

Fig. 1 Stability boundaries.

By using the functional

$$f = - \int_0^1 [dz_{,xx}^2 + gz_{,x}^2 + \mu z^2] dx$$

it is possible to show that the panel is unstable for  $M = 0$  if  $g < -\pi^2 d$ . However, the two-mode analysis of Johns and Parks,<sup>3</sup> a searching analysis of Movchan<sup>4</sup> and further work by Pritchard<sup>5</sup> all indicate that for some values of the tension less than the buckling load ( $-\pi^2 d$ ) there is a range of values of the Mach number for which the panel is stable. The accompanying results are computed for  $\mu = 200 d = \pi^2/6$  (see Fig. 1).

#### References

- <sup>1</sup> Webb, G. R. et al., "Further Study on 'A Stability Criterion for Panel Flutter via the Second Method of Liapunov,'" *AIAA Journal*, Vol. 5, No. 11, Nov. 1967, pp. 2084-2085.
- <sup>2</sup> Parks, P. C., "A Stability Criterion for Panel Flutter via the Second Method of Liapunov," *AIAA Journal*, Vol. 4, No. 1, Jan. 1966, pp. 175-177.
- <sup>3</sup> Johns, D. J. and Parks, P. C., "The Effect of Structural Damping on Panel Flutter," *Aircraft Engineering*, Vol. 32, 1960, pp. 304-308.
- <sup>4</sup> Movchan, A. A., "On the Stability of a Panel Moving in a Gas," *Prikl. Mat. i Mekh.*, Vol. 21, No. 2, 1957.
- <sup>5</sup> Pritchard, A. J., *AIAA Journal* (to be published).

## Reply by Authors to R. H. Plaut, and to P. C. Parks and A. J. Pritchard

GEORGE R. WEBB,\* BENNETT R. BASS,†  
CHARLES H. GOODMAN,† MALCOLM A. GOODMAN,†  
AND KARL M. LAND†  
Tulane University, New Orleans, La.

THE Comments by R. H. Plaut and by T. C. Parks and A. J. Pritchard are certainly correct. We appreciate their pointing out this bit of carelessness.

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\* Assistant Professor, Department of Mechanical Engineering.

† Graduate Student.

## Comment on "A Comparative Study of Numerical Methods of Elastic-Plastic Analysis"

MARTIN A. SALMON\*  
IIT Research Institute, Chicago, Ill.

PROFESSOR Marcal<sup>1</sup> claims to have shown that the initial strain method, which uses the constant strain approach, breaks down for elastic perfectly plastic materials. In order to demonstrate the claim to be false, the constant strain approach is used to obtain the solution to the symmetrical three-bar truss shown in Fig. 1.

An elastic perfectly plastic material of modulus  $E$  and yield strength  $\sigma_0$  is assumed. The area of each of the pin-connected truss members is denoted by  $A$ , the downward load at joint 1 by  $P$ , and the corresponding deflection by  $x$ . In the elastic range of material behavior the load displacement relationship in terms of the dimensionless quantities  $\bar{x} = Ex/H\sigma_0$  and  $\bar{P} = P/A\sigma_0$  is

$$(1 + a)\bar{x} = \bar{P} \quad (1)$$

where

$$a = 2 \sin^3 \phi \quad (2)$$

and where  $H$  is the depth of the truss and  $\phi$  the slope angle of the inclined truss members. Equation (1) holds for values of  $\bar{x}$  in the range  $0 \leq \bar{x} \leq 1$ , that is, for  $\bar{P} \leq 1 + a$ . For larger values of  $\bar{P}$  the center bar of the truss becomes fully plastic while the inclined bars remain elastic until the collapse load  $P_c$  is reached, where

$$\bar{P}_c = 1 + 2 \sin \phi \quad (3)$$

For values of  $\bar{P}$  in the range  $1 + a \leq \bar{P} \leq \bar{P}_c$  the load-displacement relationship can be written in the form

$$(1 + a)\bar{x} = \bar{P} + (\bar{x} - 1) \quad (4)$$

Equation (4) is the initial strain method formulation. The bracketed term on the right-hand side of Eq. (4) can be thought of as a fictitious load that, when added to the actual load, accounts for the effects of plastic flow in the vertical bar. The so-called tangent modulus formulation is obtained by eliminating the displacement from the right-hand side of Eq. (4) to give

$$a\bar{x} = \bar{P} - 1 \quad (5)$$

Solution of Eq. (4) by the following iterative scheme is called the constant strain method:

$$\bar{x}^{(n)} = \frac{1}{1 + a} [\bar{P} - 1 + \bar{x}^{(n-1)}] \quad n = 1, 2, \dots \quad (6)$$

in which  $\bar{x}^{(n)}$  is the  $n$ th iterate and  $\bar{x}^{(0)} = 1$ . An explicit expression for the value of the  $n$ th iterate can be obtained from Eq. (6) as

$$\bar{x}^{(n)} = [(\bar{P} - 1)/a][1 - 1/(1 + a)^n] + 1/(1 + a)^n \quad (7)$$

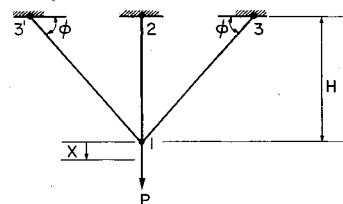


Fig. 1 Symmetrical three-bar truss.

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\* Senior Scientist, Structures Research.